

I. PART 1. MULTIPLE CHOICES QUESTIONS (7.0 points)

Write the correct answer (A, B, C or D) for each of the following questions in the correspondingly numbered space on your answer sheet.

Question 1. Given trapezoid $ABCD$ with AB and CD parallel and $\hat{A} - \hat{D} = 40^\circ, \hat{B} = 5\hat{C}$. Then $\hat{A} + \hat{B}$ is equal to

- A. 100° B. 220° C. 140° D. 260°

Question 2. The value of $A = \frac{x^3 + 27}{x + 3} - \frac{x^4 - 16}{x^2 + 4}$ with $x = \frac{1}{4}$ is equal to

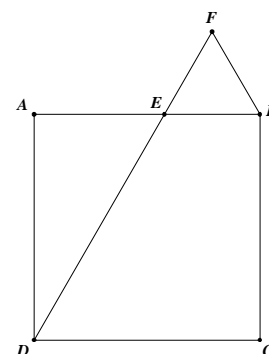
- A. $\frac{49}{4}$ B. $\frac{21}{5}$ C. $\frac{61}{4}$ D. $\frac{43}{4}$

Question 3. Which of the following statements is **false**?

- A. $(a^2 - b^2)(a + b) = a^3 - b^3$ B. $(a^2 - b^2)(a^2 + b^2) = a^4 - b^4$
C. $(2 - a)[(a + 1)^2 + 3] = 8 - a^3$ D. $(b - a)(a^2 + b^2 + ab) = b^3 - a^3$

Question 4. On the right figure, let $ABCD$ be a square, E and F are two points on AB and DE , respectively such that BEF is an equilateral triangle. If $AB = \sqrt{3}$, then EF is equal to

- A. $\frac{\sqrt{3}}{2}$ B. $\frac{\sqrt{3}}{3}$
C. $2 - \sqrt{3}$ D. $\sqrt{3} - 1$



Question 5. The quotient of the division of polynomial $2019x^{2018}y^{2020} - 3x^4y^6$ by polynomial $\frac{3}{2}x^3y^2$ is equal to

- A. $1346x^{2015}y^{2018} - 2xy^4$ B. $1346x^{2021}y^{2022} - 2x^7y^8$
C. $\frac{6057}{2}x^{2015}y^{2018} - \frac{9}{2}xy^4$ D. $\frac{6057}{2}x^{2021}y^{2022} - \frac{9}{2}x^7y^8$

Question 6. Let $Q(x) = 2x^3 + ax^2 + bx + c$ where a, b, c are given constants. If $Q(0) = 1, Q(-1) = 2, Q(2) = 3$, the value of $a + b + c$ is equal to

- A. $\frac{20}{3}$ B. -2 C. 4 D. $-\frac{14}{3}$

Question 7. What is the value of x in $\frac{-3x^2 + 2x + 1}{x^2 + 3x - 4} = 1$?

A. $x = 1$ or $x = -\frac{1}{3}$

B. $x = -\frac{5}{4}$

C. $x = -\frac{1}{3}$

D. Does not exist x

Question 8. Let $ABCD$ be a parallelogram such that $\widehat{ADB} = 90^\circ$, $AB = 5$, $AD = 3$, then AC is

A. 8

B. $2\sqrt{13}$

C. $\sqrt{13}$

D. 4

Question 9. Suppose that x and y are two integer numbers such that $x - y - xy = 79$ and $x^2 + y^2 = 130$.

The value of $(x - y + 1)^2$ is equal to

A. 260

B. 209

C. 289

D. 288

Question 10. Given expression $\frac{x+1}{x^2(x+2)} + \frac{3}{|x+2|}$. The expression is well-defined with which of the

following conditions of x ?

A. $x \neq 0$

B. $x \neq 0$ or $x \neq -2$

C. $x \neq -2$

D. $x \neq 0$ and $x \neq -2$

Question 11. The value of expression $A = x^5 - 3x^2y^3 + 3x^3y^2 - y^5$ with $x = -2$ and $y = 3$ is equal to

A. -103

B. -167

C. -815

D. -383

Question 12. Factorize the polynomial $x^4 + 2020x^2 + 2019x + 2020$.

A. $(x+1)(x^2 + x + 2020)$

B. $(x^2 + x + 1)(x^2 - x + 2020)$

C. $(x+1)^2(x^2 - x + 2020)$

D. $(x+2020)(x^3 + 2020x + 2019)$

Question 13. Let a be a positive number such that $\left|a - \frac{1}{a}\right| = 1$. The value of $a^2 + \frac{1}{a^2}$ is equal to

A. -1

B. 2

C. 1

D. 3

Question 14. Given square $ABCD$, E is mid-point of AB and F is point on BC such that $BF = 2FC$. If $AB = 6$, then EF is

A. 4

B. 7

C. 6

D. 5

Question 15. The sum of all angles in a quadrilateral is equal to

A. 540°

B. 270°

C. 360°

D. 180°

Question 16. Simplify the expression $\frac{1}{x^2 - 3x + 2} - \frac{3}{4 - 2x}$?

A. $\frac{3x-1}{2(x-1)(x-2)}$

B. $\frac{3x-1}{x^2 - 3x + 2}$

C. $\frac{5-3x}{2(x^2 - 3x + 2)}$

D. $\frac{1-3x}{2(x-1)(x-2)}$

Question 17. Let n be an integer. Suppose that $n^2 + n - 2$ is divisible by $n + 1$. What is the value of n ?

A. 1

B. 4

C. 2

D. 3

Question 18. The coefficient of x^2 in the expansion of $(1+x)(2+x^2)(3+x^3)\dots(7+x^7)$ is equal to

A. 28

B. 1

C. 5040

D. 2520

Question 19. Let $ABCD$ be an isosceles trapezoid with two bases AB , CD and AC is perpendicular to BD . The line AC meets BD at K . If $AC = 12$ and $AK = 4$ then DC is equal to

A. 8

B. $4\sqrt{2}$

C. $8\sqrt{2}$

D. 4

Question 20. Given trapezoid $ABCD$ with two bases AB , CD . Suppose that M and N are mid-points of the sides AD and BC . Let AC and BD intersect MN at I and K , respectively. If $AB = 6\text{cm}$, $CD = 8\text{cm}$ then IK is equal to

- A. 1cm B. 4cm C. 3cm D. 7cm

Question 21. Given rectangular $ABCD$ with $AB = 4$, $AD = 6$, M and N are two points on the sides AB and DC , respectively such that $AM = MB$ and $DN = 2NC$. The area of triangle DMN is equal to

- A. 6 B. 8 C. 4 D. 12

Question 22. The common denominator of $\frac{1}{(x-1)(4x^2-4x+1)}$ and $\frac{x+1}{(2x-1)(x^2-2x+1)}$ is equal to

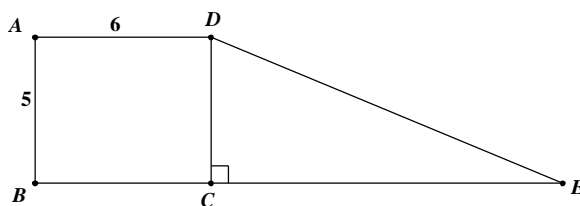
- A. $(x-1)^2(1-2x)^2$ B. $(2x-1)(4x^2-4x+1)$ C. $(2x-1)(x-1)$ D. $(x-1)^2(2x-1)$

Question 23. Simplify the expression $\left(\frac{1-x^3}{1-x} - x\right) : \left(\frac{1-x}{1+x^2}\right)$?

- A. $\frac{x^2+x+1}{1+x^2}$ B. $1-x$ C. $\frac{1+x^2}{1-x}$ D. $\frac{(1+x^2)^2}{1-x}$

Question 24. On the right figure, given rectangle $ABCD$ and right triangle DCE which have the same area. The height DE is equal to

- A. $5\sqrt{2}$ B. 13
C. $\sqrt{125}$ D. 12



Question 25. What is the solution of $A = 2x^5 - \frac{1}{2}x^3 + \frac{3}{4}x^2 - 1$ with $x = -2$?

- A. 64 B. 58 C. -58 D. -64

Question 26. Given the lengths of sides of a rectangular are 6cm and 8cm , respectively. What is the length of the diagonal?

- A. 8cm B. 12cm C. 6cm D. 10cm

Question 27. Calculate the sum of all coefficients in the expansion of expression $(x^2 - 2x - 3)^7$.

- A. -2187 B. -4 C. -16384 D. -78125

Question 28. Let a, b be a real numbers such that $\frac{2x+1}{x^2+6x+9} = \frac{a}{x+3} + \frac{b}{(x+3)^2}$. The value of $a+b$ is equal to

- A. -5 B. -3 C. 1 D. 7

Question 29. A square is inscribed inside a rhombus with diagonals 6 and 10. Find the area of the square.

- A. $\frac{15}{8}$ B. $\frac{225}{16}$ C. $\frac{15}{4}$ D. $\frac{225}{64}$

Question 30. Given triangle ABC with the right angle A . Let M be a point on BC such that $BM = 2MC$. If $AB = 3\text{cm}$, $AC = 4\text{cm}$ then the area of triangle ACM is equal to

- A. 2cm^2 B. 3cm^2 C. 6cm^2 D. 4cm^2

Question 31. Suppose that polynomial $x^4 + 2x^3 + x^2 + m$ is divisible by polynomial $x^2 + x - 1$. The value of m is

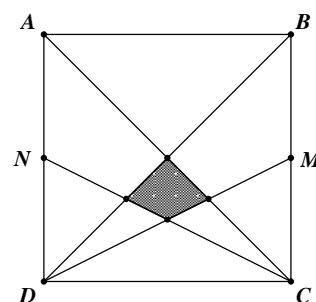
- A. -2 B. 4 C. -1 D. 1

Question 32. Given trapezoid $ABCD$ with two bases $AB = 3\text{cm}$, $CD = 13\text{cm}$. Suppose that the area of the trapezoid is 48cm^2 . The height AH is equal to

- A. 3cm B. $1,5\text{cm}$ C. 24cm D. 6cm

Question 33. On the right figure, given square $ABCD$ with $AB = 10$. Assume that M and N are mid-points of the sides AD and BC , respectively. The area of the black space is equal to

- A. 5 B. $\frac{25}{12}$
C. $\frac{25}{6}$ D. $\frac{25}{7}$



Question 34. Find the minimum value of the expression $2x^2 - 4|x - 2| - 8x + 3$.

- A. -7 B. 3 C. -5 D. -6

Question 35. Quadrilateral $ABCD$ is a rhombus with perimeter 48cm . The length of the side of rhombus is equal to

- A. 16cm B. 9cm C. 12cm D. 24cm

II. PART II. PROBLEM SOLVING (3.0 points)

Write the solutions to the following problems in the provided space on your answer sheet.

Problem 1. Find all prime numbers p such that $16p + 1$ is a perfect cube.

Problem 2. A pair of numbers are *twin primes* if they differ by two and both are primes. Prove that, except the pair $\{3; 5\}$, the sum of any pair of twin primes is a multiple of 12.

Problem 3. In rectangle $ABCD$, the length of side AB is twice as the length of side BC . A point P is taken on side AB such that $BP = \frac{1}{4}AB$. Show that BD is perpendicular to CP .

-THE END-

Student's full name: Student's ID:

First observer's name and signature: Second observer's name and signature: